

APPLIED MATHEMATICS COLLOQUIUM

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A moment matrix approach to symmetric cubatures

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Abstract:

A quadrature is an approximation of the definite integral of a function by a weighted sum of function values at specified points, or nodes, within the domain of integration. Gaussian quadratures are constructed to yield exact results for any polynomials of degree $2r-1$ or less by a suitable choice of r nodes and weights. Cubature is a generalization of quadrature in higher dimension. Constructing a cubature amounts to find a linear form $p \rightarrow a_1 p(x_1) + \dots + a_r p(x_r)$ from the knowledge of its restriction to polynomials of degree d or less. The unknowns are the weights a_j and the nodes x_j .

An approach based on moment matrices was proposed in [2,4]. We give a basis-free version in terms of the Hankel operator H associated to a linear form. The existence of a cubature of degree d with r nodes boils down to conditions of ranks and positive semidefiniteness on H . We then recognize the nodes as the solutions of a generalized eigenvalue problem.

Standard domains of integration are symmetric under the action of a finite group. It is natural to look for cubatures that respect this symmetry [1,3]. Introducing adapted bases obtained from representation theory, the symmetry constraint allows to block diagonalize the Hankel operator H . The size of the blocks is explicitly related to the orbit types of the nodes. From the computational point of view, we then deal with smaller-sized matrices both for securing the existence of the cubature and computing the nodes.